

### Amendments to the Claims

This listing of claims will replace all prior versions, and listings, of claims in the application:

#### Listing of Claims:

1-2. (Canceled)

3. (Previously presented) A cryptographic communications method for communications of information between entities wherein a plurality of centers are provided, each of which generates secret keys peculiar to the entities using divided pieces of information resulting from division of information specifying each of the entities, the divided information used to generate the secret keys allowing diminished sizes of the secret keys; one entity generates a first common key using a first component contained in at least one secret key generated by at least one of the plurality of centers, the secret key being peculiar to the one entity, encrypts plaintext to ciphertext using the first common key and sends the ciphertext to another entity, the first component corresponding to one or more of the divided pieces of information specifying said another entity; and said another entity generates a second common key identical to the first common key using a second component contained in secret keys peculiar to the another entity sent from said centers, and decrypts said ciphertext to the original plaintext using the second common key, the second component corresponding to one or more of the divided pieces of information specifying the one entity.

4. (Previously presented) A cryptographic communications method for communicating information between entities wherein:

secret keys peculiar to said entities are sent from a center to said entities;

one entity encrypts plaintext to ciphertext using a first common key derived from a first secret key peculiar to the one entity sent from said center and sends the ciphertext to another entity;

said another entity decrypts said ciphertext to the original plaintext using a second common key identical to the first common key, the second common key being

derived from a second secret key peculiar to said another entity sent from said center, characterized in that;

a plurality of said centers are deployed;

each of said plurality of centers generates secret keys peculiar to said entities by adding random numbers peculiar to said entities to divided pieces of information resulting from division of information specifying each of said entities, the divided information used to generate the secret keys allowing diminished sizes of the secret keys; and

each of said entities generates a common key using a component, contained in the secret key peculiar to that selfsame entity, corresponding to one or more of the divided pieces of information obtained from each of said plurality of centers which specify an opposite entity.

5. (Original) The cryptographic communications method according to claim 4, wherein computation formulas for generating secret keys at said centers are as follows:

$$\begin{aligned}\vec{S}_{i1} &\equiv g^{\alpha_{i1} H_1} [\vec{I}_{i1}] \pmod{P} \\ \vec{S}_{i2} &\equiv \alpha_{i2} H_2 [\vec{I}_{i2}] \pmod{P-1} \\ &\vdots \\ \vec{S}_{iK} &\equiv \alpha_{iK} H_K [\vec{I}_{iK}] \pmod{P-1}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th piece of divided information specifying entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th piece of divided information specifying entity  $i$ ;

$P$  is a prime number;

$K$  is number of divisions in the information specifying entity  $i$ ;

$g$  is primitive element for GF (P);  
 $H_j$  is a symmetrical  $2^M \times 2^M$  matrix made up of random numbers;  
 $M$  is size of divisions in the information specifying entity  $i$ ; and  
 $\alpha_{ij}$  is a personal secret random number for entity  $i$  (where  $\alpha_{i1} \dots \alpha_{iK} \equiv 1 \pmod{P-1}$ )).

6. (Original) The cryptographic communications method according to claim 5, wherein computation formulas for generating common keys at said entities are as follows:

$$\begin{aligned} K_{im} &\equiv \overrightarrow{S_{i1}} [\overrightarrow{I_{m1}}] \overrightarrow{S_{i2}} [\overrightarrow{I_{m2}}] \dots \overrightarrow{S_{iK}} [\overrightarrow{I_{mK}}] \\ &\equiv g^{\overrightarrow{\alpha_{i1}} \dots \overrightarrow{\alpha_{iK}}} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] \dots H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}] \\ &\equiv g^{H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] \dots H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}]} \pmod{P} \end{aligned}$$

where

$K_{im}$  is common key generated by one entity  $i$  for another entity  $m$ ; and  
vector  $s_{ij}$  [vector  $I_{ij}$ ] is a component contained in secret key vector  $s_{ij}$  of entity  $i$ , corresponding to divided piece of information specifying entity  $m$ .

7. (Canceled)

8. (Previously presented) A cryptographic communications system for reciprocally performing, between a plurality of entities, encrypting processing for encrypting plaintext that is information to be sent into ciphertext and decrypting processing for decrypting ciphertext so sent back into original plaintext; comprising:

a plurality of centers that generate secret keys peculiar to said entities using pieces of information resulting from division of information specifying each of said entities and that sends said secret keys to said entities, the divided information used to generate the secret keys allowing diminished sizes of the secret keys; and

a plurality of entities each of which generates a common key employed mutually in said encryption and decryption processing when communicating with another entity, using a component corresponding to a divided specified information to each entity, contained in own secret key sent from the centers, the component further corresponding to one or more pieces of information specifying said another entity.

9-12. (Canceled)

13. (Previously presented) A cryptographic communications method for communications of information between entities wherein a plurality of centers are provided, each of which generates secret keys peculiar to the entities using divided specifying information resulting from division of information specifying each of the entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret keys; one entity generates a first common key using a first component contained in secret keys peculiar to the one entity sent from the centers, encrypts plaintext to ciphertext using the first common key and sends the ciphertext to another entity, the first component corresponding to one or more of the divided pieces of information specifying said another entity; and said another entity generates a second common key identical to the first common key using a second component contained in secret keys peculiar to the another entity sent from said centers, and decrypts said ciphertext to the original plaintext using the second common key, the second component corresponding to one or more of the divided pieces of information specifying the one entity; secret keys for first block of divided specifying information have a multi-layer structure; and secret keys for remaining blocks of divided specifying information have a single-layer structure.

14. (Previously presented) A secret key generation method for generating secret keys peculiar to entities using divided specifying information resulting from division of information specifying said entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret

keys, each entity generating a common key by using a component corresponding to the divided specifying information of another entity, wherein:

computation formulas for generating said secret keys are as follows:

$$\begin{aligned}
 \vec{S}_{i1} &= \alpha_i H_1 [\vec{I}_{i1}] + \beta_{i1} \vec{1} \\
 \vec{S}_{i2} &= \alpha_i H_2 [\vec{I}_{i2}] + \beta_{i2} \vec{1} \\
 &\vdots \\
 \vec{S}_{ij} &= \alpha_i H_j [\vec{I}_{ij}] + \beta_{ij} \vec{1} \\
 &\vdots \\
 \vec{S}_{iK} &= \alpha_i H_K [\vec{I}_{iK}] + \beta_{iK} \vec{1} \\
 \vec{g}_{i0} &\equiv g^{\alpha_i^{-T}} \vec{1} \pmod{N} \\
 \vec{g}_{i1} &\equiv g^{\alpha_i^{-T}} \vec{S}_{i1} \pmod{N} \\
 \vec{g}_{i2} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^2 \pmod{N} \\
 &\vdots \\
 \vec{g}_{it} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^t \pmod{N} \\
 &\vdots \\
 \vec{g}_{iT} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^T \pmod{N}
 \end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $\vec{1}$  is a vector of dimension  $K$  wherein all components are 1;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for 1st block of specifying information for entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, then the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

(ii)  $B = (b_{\mu\nu})$

(iii)  $b_{\mu\nu} = c^{a_{\mu\nu}}$

(iv)  $b_{\mu\nu} = a_{\mu\nu}^c$

15. (Previously presented) An encryption method wherein:

secret keys peculiar to entities are generated using divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret keys, each entity generating a common key by using a component corresponding to the divided specifying information of another entity;

plaintext is encrypted to ciphertext at one entity using a common key generated using a component contained in the secret key peculiar to the one entity, the component corresponding to divided specifying information for another entity that is a destination of said ciphertext; and

computation formulas for generating said secret keys peculiar to said entities are as follows:

$$\begin{aligned}\vec{S}_{i1} &= \alpha_i H_1[\vec{I}_{i1}] + \beta_{i1} \vec{I} \\ \vec{S}_{i2} &= \alpha_i H_2[\vec{I}_{i2}] + \beta_{i2} \vec{I} \\ &\vdots \\ \vec{S}_{ij} &= \alpha_i H_j[\vec{I}_{ij}] + \beta_{ij} \vec{I} \\ &\vdots \\ \vec{S}_{iK} &= \alpha_i H_K[\vec{I}_{iK}] + \beta_{iK} \vec{I}\end{aligned}$$

$$\begin{aligned}\vec{g}_{i0} &\equiv g \alpha_i^{-T} \vec{1} \pmod{N} \\ \vec{g}_{i1} &\equiv g \alpha_i^{-T} \vec{S}_{i1} \pmod{N} \\ \vec{g}_{i2} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^2 \pmod{N} \\ &\vdots \\ \vec{g}_{it} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^t \pmod{N} \\ &\vdots \\ \vec{g}_{iT} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^T \pmod{N}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are  $1$ ;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$

(where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$

(where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for 1st block of specifying information for entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

(ii)  $B = (b_{\mu\nu})$

$$(iii) \quad b_{\mu\nu} = c_{\mu\nu}^a$$

$$(iv) \quad b_{\mu\nu} = a_{\mu\nu}^c$$

16. (Original) The encryption method according to claim 15, wherein computation formulas for generating said common keys are as follows:

$$g_{0im} = \overrightarrow{g_{i0}} [\overrightarrow{I_{m1}}]$$

$$g_{1im} = \overrightarrow{g_{i1}} [\overrightarrow{I_{m1}}]$$

$$g_{tim} = \overrightarrow{g_{it}} [\overrightarrow{I_{m1}}]$$

$$g_{Tim} = \overrightarrow{g_{iT}} [\overrightarrow{I_{m1}}]$$

$$x_{2im} = \overrightarrow{s_{i2}} [\overrightarrow{I_{m2}}]$$

$$x_{jim} = \overrightarrow{s_{ij}} [\overrightarrow{I_{mj}}]$$

$$x_{Kim} = \overrightarrow{s_{iK}} [\overrightarrow{I_{mK}}]$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T g_{tim}^{C_t y_{im}^{(T-t)}} \\ &\equiv g_i^{-T} \sum_{t=0}^T C_t x_{tim}^t y_{im}^{T-t} \\ &\equiv g_i^{-T} (\alpha_{im} + y_{im})^T \\ &\equiv g_i^{-T} (\alpha_{im} + x_{kim})^T \\ &\equiv g_i^{-T} (\alpha_i H_1(\overrightarrow{I_{i1}}) [\overrightarrow{I_{m1}}] + \beta_{i1} + \dots + \alpha_i H_K(\overrightarrow{I_{iK}}) [\overrightarrow{I_{mK}}] + \beta_{iK})^T \\ &\equiv g_i^{-T} (\alpha_i H_1(\overrightarrow{I_{i1}}) [\overrightarrow{I_{m1}}] + \dots + H_K(\overrightarrow{I_{iK}}) [\overrightarrow{I_{mK}}]) + \lambda \infty)^T \\ &\equiv g_i^{-T} (\alpha_i H_1(\overrightarrow{I_{i1}}) [\overrightarrow{I_{m1}}] + \dots + H_K(\overrightarrow{I_{iK}}) [\overrightarrow{I_{mK}}])^T \\ &\equiv g_i^{-T} (H_1(\overrightarrow{I_{i1}}) [\overrightarrow{I_{m1}}] + \dots + H_K(\overrightarrow{I_{iK}}) [\overrightarrow{I_{mK}}])^T \pmod{N} \end{aligned}$$

where

$g_{tim}$  (= vector  $g_{it}$  [vector  $I_{m1}$ ]) is a component corresponding to vector  $I_{m1}$  for entity  $m$ , selected from own vector  $g_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );



$x_{lim} = \text{vector } s_{il} [\text{vector } I_{ml}]$ ;

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .

17. (Previously presented) A cryptographic communications method for communications of information between entities, wherein

a plurality of centers are deployed, each of which generates secret keys peculiar to said entities using divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret keys, and sends the secret keys to the entities respectively;

one entity generates a first common key using a first component contained in secret keys peculiar to the one entity sent from the centers, encrypts plaintext to ciphertext using the first common key, and sends the ciphertext to said another entity, the first component corresponding to divided specifying information for another entity;

said another entity generates a second common key identical to the first common key using a second component contained in secret keys peculiar to said another entity sent from the centers, and decrypts said ciphertext using the second common key, the second component corresponding to divided specifying information for the one entity; and

computation formulas for generating said secret keys at said centers are as follows:

$$\vec{S}_{i1} = \alpha_i H_1 [\vec{I}_{i1}] + \beta_{i1} \vec{1}$$

$$\vec{S}_{i2} = \alpha_i H_2 [\vec{I}_{i2}] + \beta_{i2} \vec{1}$$

$$\vec{S}_{ij} = \alpha_i H_j [\vec{I}_{ij}] + \beta_{ij} \vec{1}$$

$$\vec{S}_{iK} = \alpha_i H_K [\vec{I}_{iK}] + \beta_{iK} \vec{1}$$

$$\begin{aligned}\vec{g}_{i0} &\equiv g^{\alpha_i^{-T}} \vec{1} \pmod{N} \\ \vec{g}_{i1} &\equiv g^{\alpha_i^{-T}} \vec{S}_{i1} \pmod{N} \\ \vec{g}_{i2} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^2 \pmod{N} \\ \vdots & \\ \vec{g}_{it} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^t \pmod{N} \\ \vdots & \\ \vec{g}_{iT} &\equiv g^{\alpha_i^{-T}} \langle \vec{S}_{i1} \rangle^T \pmod{N}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are  $1$ ;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

(ii)  $B = (b_{\mu\nu})$

(iii)  $b_{\mu\nu} = c^{a_{\mu\nu}}$

(iv)  $b_{\mu\nu} = a_{\mu\nu}^c$

18. (Original) The cryptographic communications method according to claim 17, wherein computation formulas for generating said common keys are as follows:

$$\begin{aligned} \vec{g}_{0im} &= \vec{g}_{i0} [\vec{I}_{m1}] \\ \vec{g}_{1im} &= \vec{g}_{i1} [\vec{I}_{m1}] \\ &\vdots \\ \vec{g}_{tim} &= \vec{g}_{it} [\vec{I}_{m1}] \\ &\vdots \\ \vec{g}_{Tim} &= \vec{g}_{iT} [\vec{I}_{m1}] \\ \vec{x}_{2im} &= \vec{s}_{i2} [\vec{I}_{m2}] \\ &\vdots \\ \vec{x}_{jim} &= \vec{s}_{ij} [\vec{I}_{mj}] \\ &\vdots \\ \vec{x}_{kim} &= \vec{s}_{iK} [\vec{I}_{mK}] \end{aligned}$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T \vec{g}_{tim}^{T C_t y_{im}^{(T-t)}} \\ &\equiv \vec{g}_{i1}^{-T} \sum_{t=0}^T C_t x_{tim}^t y_{im}^{T-t} \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{im} + y_{im})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{im} + \dots + x_{kim})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_i H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \beta_{i1} + \dots + \alpha_i H_K [\vec{I}_{iK}] [\vec{I}_{mK}] + \beta_{iK})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_i (H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]) + \lambda_{i0})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_i (H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]))^T \\ &\equiv \vec{g}_{i1}^{H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]} \pmod{N} \end{aligned}$$

where

$\vec{g}_{tim}$  (= vector  $\vec{g}_{it}$  [vector  $\vec{I}_{m1}$ ]) is a component corresponding to vector  $\vec{I}_{m1}$  for entity  $m$ , selected from own vector  $\vec{g}_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$\vec{x}_{lim}$  = vector  $\vec{s}_{il}$  [vector  $\vec{I}_{ml}$ ];

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .

19. (Previously presented) A common key generator provided at entities in a cryptographic communications system for generating a common key to be used in processing to encrypt plaintext to ciphertext and in processing to decrypt ciphertext back to plaintext, comprising:

storage means for storing secret keys peculiar to said entities produced, according to computation formulas given below, for divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret keys, each entity generating a common key by using a component corresponding to the divided specifying information of another entity;

selection means for selecting components corresponding to divided specifying information for opposite entities to be communicated with, from the secret keys stored; and

means for generating said common keys, according to computation formulas given below, using said components so selected:

$$\begin{aligned}\vec{s}_{i1} &= \alpha_i H_1[\vec{I}_{i1}] + \beta_{i1} \vec{1} \\ \vec{s}_{i2} &= \alpha_i H_2[\vec{I}_{i2}] + \beta_{i2} \vec{1} \\ &\vdots \\ \vec{s}_{ij} &= \alpha_i H_j[\vec{I}_{ij}] + \beta_{ij} \vec{1} \\ &\vdots \\ \vec{s}_{iK} &= \alpha_i H_K[\vec{I}_{iK}] + \beta_{iK} \vec{1}\end{aligned}$$

$$\begin{aligned}\overrightarrow{g_{i0}} &\equiv g^{\alpha_i^{-T}} \overrightarrow{1} \pmod{N} \\ \overrightarrow{g_{i1}} &\equiv g^{\alpha_i^{-T}} \overrightarrow{S_{i1}} \pmod{N} \\ \overrightarrow{g_{i2}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^2 \pmod{N} \\ &\vdots \\ \overrightarrow{g_{it}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^t \pmod{N} \\ &\vdots \\ \overrightarrow{g_{iT}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^T \pmod{N}.\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are 1;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

$$(ii) \quad B = (b_{\mu\nu})$$

$$(iii) \quad b_{\mu\nu} = c^a_{\mu\nu}$$

$$(iv) \quad b_{\mu\nu} = a_{\mu\nu}^c$$

$$g_{0im} = \overrightarrow{g_{i0}} [\overrightarrow{I_{m1}}]$$

$$g_{1im} = \overrightarrow{g_{i1}} [\overrightarrow{I_{m1}}]$$

$$g_{tim} = \overrightarrow{g_{it}} [\overrightarrow{I_{m1}}]$$

$$g_{Tim} = \overrightarrow{g_{iT}} [\overrightarrow{I_{m1}}]$$

$$x_{2im} = \overrightarrow{s_{i2}} [\overrightarrow{I_{m2}}]$$

$$x_{jim} = \overrightarrow{s_{ij}} [\overrightarrow{I_{mj}}]$$

$$x_{Kim} = \overrightarrow{s_{iK}} [\overrightarrow{I_{mK}}]$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T g_{tim}^{T-t} y_{im}^{(T-t)} \\ &\equiv g_{i1}^{a_i T} \sum_{t=0}^T c_{x_{1im}}^t y_{im}^{T-t} \\ &\equiv g_{i1}^{a_i T} (\alpha_{1im} + y_{im})^T \\ &\equiv g_{i1}^{a_i T} (\alpha_{1im} + \dots + x_{kim})^T \\ &\equiv g_{i1}^{a_i T} (\alpha_{iH_1} [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \beta_{i1} + \dots + \alpha_{iH_K} [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}] + \beta_{iK})^T \\ &\equiv g_{i1}^{a_i T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}] + \lambda \cdot 0)^T \\ &\equiv g_{i1}^{a_i T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}])^T \\ &\equiv g_{i1}^{a_i T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}])^T \pmod{N} \end{aligned}$$

where

$g_{tim}$  (= vector  $g_{it}$  [vector  $I_{m1}$ ]) is a component corresponding to vector  $I_{m1}$  for entity  $m$ , selected from own vector  $g_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$x_{lim}$  = vector  $s_{il}$  [vector  $I_{ml}$ ];

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .

20. (Previously presented) A cryptographic communications system for reciprocally performing, between a plurality of entities, encryption processing for encrypting plaintext that is information to be sent into ciphertext and decryption processing for decrypting ciphertext so sent back into original plaintext, comprising:

a plurality of centers each of which generates secret keys peculiar to said entities, according to computation formulas given below, using divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information used to generate the secret keys allowing diminished sizes of the secret keys, and sends said secret keys to said entities; and

a plurality of entities each of which generates a common key mutually employed in said encryption and decryption processing when communicating with another entity, according to computation formulas given below, using a component contained in own secret key sent from said centers, the component corresponding to divided specifying information for said another entity:

$$\begin{aligned}\vec{S}_{i1} &= \alpha_1 H_1[\vec{I}_{i1}] + \beta_{i1} \vec{I} \\ \vec{S}_{i2} &= \alpha_2 H_2[\vec{I}_{i2}] + \beta_{i2} \vec{I} \\ &\vdots \\ \vec{S}_{ij} &= \alpha_j H_j[\vec{I}_{ij}] + \beta_{ij} \vec{I} \\ &\vdots \\ \vec{S}_{iK} &= \alpha_K H_K[\vec{I}_{iK}] + \beta_{iK} \vec{I}\end{aligned}$$

$$\begin{aligned}\vec{g}_{i0} &\equiv g \alpha_i^{-T} \vec{1} \pmod{N} \\ \vec{g}_{i1} &\equiv g \alpha_i^{-T} \vec{s}_{i1} \pmod{N} \\ \vec{g}_{i2} &\equiv g \alpha_i^{-T} \langle \vec{s}_{i1} \rangle^2 \pmod{N} \\ &\vdots \\ \vec{g}_{it} &\equiv g \alpha_i^{-T} \langle \vec{s}_{i1} \rangle^t \pmod{N} \\ &\vdots \\ \vec{g}_{iT} &\equiv g \alpha_i^{-T} \langle \vec{s}_{i1} \rangle^T \pmod{N}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are  $1$ ;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for  $1$ st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

(ii)  $B = (b_{\mu\nu})$

(iii)  $b_{\mu\nu} = c^{a_{\mu\nu}}$

(iv)  $b_{\mu\nu} = a_{\mu\nu}^c$



$$\begin{aligned} \vec{g}_{0im} &= \vec{g}_{i0} [\vec{I}_{m1}] \\ \vec{g}_{1im} &= \vec{g}_{i1} [\vec{I}_{m1}] \\ &\vdots \\ \vec{g}_{tim} &= \vec{g}_{it} [\vec{I}_{m1}] \\ &\vdots \\ \vec{g}_{Tim} &= \vec{g}_{iT} [\vec{I}_{m1}] \\ \vec{x}_{2im} &= \vec{s}_{i2} [\vec{I}_{m2}] \\ &\vdots \\ \vec{x}_{jim} &= \vec{s}_{ij} [\vec{I}_{mj}] \\ &\vdots \\ \vec{x}_{Kim} &= \vec{s}_{iK} [\vec{I}_{mK}] \end{aligned}$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T \vec{g}_{tim}^{T C_t y_{im}^{(T-t)}} \\ &\equiv \vec{g}_{i1}^{a_1 T} \sum_{t=0}^T C_t x_{1im}^t y_{im}^{T-t} \\ &\equiv \vec{g}_{i1}^{a_1 T} (x_{1im} + y_{im})^T \\ &\equiv \vec{g}_{i1}^{a_1 T} (x_{1im} + \dots + x_{Kim})^T \\ &\equiv \vec{g}_{i1}^{a_1 T} (a_1 H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \beta_{i1} + \dots + a_i H_K [\vec{I}_{iK}] [\vec{I}_{mK}] + \beta_{iK})^T \\ &\equiv \vec{g}_{i1}^{a_1 T} (a_1 H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]) + \lambda \infty)^T \\ &\equiv \vec{g}_{i1}^{a_1 T} (a_1 H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}])^T \\ &\equiv \vec{g}_{i1}^{a_1 T} (H_1 [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}])^T \pmod{N} \end{aligned}$$

where

$\vec{g}_{tim}$  (= vector  $\vec{g}_{it}$  [vector  $\vec{I}_{m1}$ ]) is a component corresponding to vector  $\vec{I}_{m1}$  for entity  $m$ , selected from own vector  $\vec{g}_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$x_{lim} = \text{vector } s_{il} [\text{vector } I_{ml}];$

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .

21. (Previously presented) A computer readable recording medium embodied in a tangible physical object for storing a program that generates at entities involved in communications a common key mutually used in processing to encrypt plaintext to ciphertext and in processing to decrypt said ciphertext back to said plaintext in a cryptographic communications system, comprising:

first program code means for causing said computer to select a component corresponding to divided specifying information of one entity that is a ciphertext recipient from a secret key peculiar to another entity that is a ciphertext sender, according to computation formulas given below, for each of divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information allowing a diminished size of the secret key; and

second program code means for causing said computer to generate said common key, according to computation formulas given below, using said components selected:

$$\begin{aligned}\vec{S}_{i1} &= \alpha_i H_1 [\vec{I}_{i1}] + \beta_{i1} \vec{1} \\ \vec{S}_{i2} &= \alpha_i H_2 [\vec{I}_{i2}] + \beta_{i2} \vec{1} \\ &\vdots \\ \vec{S}_{ij} &= \alpha_i H_j [\vec{I}_{ij}] + \beta_{ij} \vec{1} \\ &\vdots \\ \vec{S}_{iK} &= \alpha_i H_K [\vec{I}_{iK}] + \beta_{iK} \vec{1}\end{aligned}$$

$$\begin{aligned}\overrightarrow{g_{i0}} &\equiv g^{\alpha_i^{-T}} \overrightarrow{1} \pmod{N} \\ \overrightarrow{g_{i1}} &\equiv g^{\alpha_i^{-T}} \overrightarrow{S_{i1}} \pmod{N} \\ \overrightarrow{g_{i2}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^2 \pmod{N} \\ \vdots & \\ \overrightarrow{g_{it}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^t \pmod{N} \\ \vdots & \\ \overrightarrow{g_{iT}} &\equiv g^{\alpha_i^{-T}} \langle \overrightarrow{S_{i1}} \rangle^T \pmod{N}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are  $1$ ;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

$$(ii) \quad B = (b_{\mu\nu})$$

$$(iii) \quad b_{\mu\nu} = c_{\mu\nu}^a$$

$$(iv) \quad b_{\mu\nu} = a_{\mu\nu}^c$$

$$g_{0im} = \overrightarrow{g_{i0}} [\overrightarrow{I_{m1}}]$$

$$g_{1im} = \overrightarrow{g_{i1}} [\overrightarrow{I_{m1}}]$$

$$g_{tim} = \overrightarrow{g_{it}} [\overrightarrow{I_{m1}}]$$

$$g_{Tim} = \overrightarrow{g_{iT}} [\overrightarrow{I_{m1}}]$$

$$x_{2im} = \overrightarrow{s_{i2}} [\overrightarrow{I_{m2}}]$$

$$x_{jim} = \overrightarrow{s_{ij}} [\overrightarrow{I_{mj}}]$$

$$x_{kim} = \overrightarrow{s_{iK}} [\overrightarrow{I_{mK}}]$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T g_{tim}^{T C_t y_{im}^{(T-t)}} \\ &\equiv g_{i1}^{-T} \sum_{t=0}^T C_t x_{1im}^t y_{im}^{T-t} \\ &\equiv g_{i1}^{-T} (\alpha_{1im} + y_{im})^T \\ &\equiv g_{i1}^{-T} (\alpha_{1im} + \dots + x_{kim})^T \\ &\equiv g_{i1}^{-T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \beta_{i1} + \dots + \alpha_{iK} H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}] + \beta_{iK})^T \\ &\equiv g_{i1}^{-T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}]) + \lambda \infty)^T \\ &\equiv g_{i1}^{-T} (\alpha_{i1} H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}])^T \\ &\equiv g_{i1}^{-T} (H_1 [\overrightarrow{I_{i1}}] [\overrightarrow{I_{m1}}] + \dots + H_K [\overrightarrow{I_{iK}}] [\overrightarrow{I_{mK}}])^T \pmod{N} \end{aligned}$$

where

$g_{tim}$  (= vector  $g_{it}$  [vector  $I_{m1}$ ]) is a component corresponding to vector  $I_{m1}$  for entity  $m$ , selected from own vector  $g_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$x_{lim}$  = vector  $s_{il}$  [vector  $I_{ml}$ ];

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .

22. (Canceled)

23. (Previously presented) A computer readable program embodied in a tangible physical object for generating at entities involved in communications a common key mutually used in processing to encrypt plaintext to ciphertext and in processing to decrypt said ciphertext back to said plaintext in a cryptographic communications system, comprising:

first code segment for causing a computer to select a component corresponding to divided specifying information of one entity that is a ciphertext recipient from a secret key peculiar to another entity that is a ciphertext sender, according to computation formulas given below, for each of divided specifying information resulting from division of information specifying each of said entities into a plurality of blocks, the divided information allowing a diminished size of the secret key; and

second code segment for causing said computer to generate said common key, according to computation formulas given below, wherein said computer generates said common keys by using a component corresponding to the divided specifying information of another computer, using said components selected:

$$\begin{aligned}\vec{S}_{i1} &= \alpha H_1[\vec{I}_{i1}] + \beta_{i1} \vec{1} \\ \vec{S}_{i2} &= \alpha H_2[\vec{I}_{i2}] + \beta_{i2} \vec{1} \\ &\vdots \\ \vec{S}_{ij} &= \alpha H_j[\vec{I}_{ij}] + \beta_{ij} \vec{1} \\ &\vdots \\ \vec{S}_{iK} &= \alpha H_K[\vec{I}_{iK}] + \beta_{iK} \vec{1}\end{aligned}$$

$$\begin{aligned}\vec{g}_{i0} &\equiv g \alpha_i^{-T} \vec{1} \pmod{N} \\ \vec{g}_{i1} &\equiv g \alpha_i^{-T} \vec{S}_{i1} \pmod{N} \\ \vec{g}_{i2} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^2 \pmod{N} \\ \vdots & \\ \vec{g}_{it} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^t \pmod{N} \\ \vdots & \\ \vec{g}_{iT} &\equiv g \alpha_i^{-T} \langle \vec{S}_{i1} \rangle^T \pmod{N}\end{aligned}$$

where

vector  $s_{ij}$  is a secret key corresponding to  $j$ 'th divided specifying information for entity  $i$  ( $j = 1, 2, \dots, K$ )

[vector  $I_{ij}$ ] is  $j$ 'th divided specifying information for entity  $i$ ;

vector  $1$  is a vector of dimension  $K$  wherein all components are  $1$ ;

$H_j$  is a symmetrical  $2^{M_j} \times 2^{M_j}$  matrix made up of random numbers;

$M_j$  is size of  $j$ 'th divided specifying information for entity  $i$ ;

$K$  is number of block divisions in information specifying entity  $i$ ;

$\alpha_i$  is a personal secret random number for entity  $i$  (where  $\gcd(\alpha_i, \lambda(N)) = 1$  and  $\lambda(\bullet)$  is Carmichael function);

$N$  is such that  $N = PQ$  (where  $P$  and  $Q$  are prime);

$\beta_{ij}$  is a personal secret random number for entity  $i$  (where  $\beta_{i1} + \beta_{i2} + \dots + \beta_{iK} = \lambda(N)$ );

$g$  is maximum generating element with modulo  $N$ ;

vector  $g_{it}$  is a secret key for  $1$ st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$T$  is degree of exponent portion; and

if  $c$  is a scalar, and  $A$  and  $B$  are matrixes represented in (i) and (ii) below, the expressions  $B = c^A$  and  $B = \langle A \rangle^c$  represent (iii) and (iv) below, respectively.

(i)  $A = (a_{\mu\nu})$

(ii)  $B = (b_{\mu\nu})$

$$(iii) \quad b_{\mu\nu} = c^a_{\mu\nu}$$

$$(iv) \quad b_{\mu\nu} = a_{\mu\nu}^c$$

$$\vec{g}_{0im} = \vec{g}_{i0} [\vec{I}_{m1}]$$

$$\vec{g}_{1im} = \vec{g}_{i1} [\vec{I}_{m1}]$$

$$\vdots$$

$$\vec{g}_{tim} = \vec{g}_{it} [\vec{I}_{m1}]$$

$$\vdots$$

$$\vec{g}_{Tim} = \vec{g}_{iT} [\vec{I}_{m1}]$$

$$\vec{x}_{2im} = \vec{s}_{i2} [\vec{I}_{m2}]$$

$$\vdots$$

$$\vec{x}_{jim} = \vec{s}_{ij} [\vec{I}_{mj}]$$

$$\vdots$$

$$\vec{x}_{kim} = \vec{s}_{iK} [\vec{I}_{mK}]$$

$$\begin{aligned} K_{im} &\equiv \prod_{t=0}^T \vec{g}_{tim}^{T C_t y_{im}^{(T-t)}} \\ &\equiv \vec{g}_{i1}^{-T} \sum_{t=0}^T C_t x_{tim} \cdot y_{im}^{T-t} \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{1im} + y_{im})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{1im} + \dots + x_{kim})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{iH_1} [\vec{I}_{i1}] [\vec{I}_{m1}] + \beta_{i1} + \dots + \alpha_{iH_K} [\vec{I}_{iK}] [\vec{I}_{mK}] + \beta_{iK})^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{iH_1} [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]) + \lambda \otimes )^T \\ &\equiv \vec{g}_{i1}^{-T} (\alpha_{iH_1} [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}])^T \\ &\equiv \vec{g}_{iH_1} [\vec{I}_{i1}] [\vec{I}_{m1}] + \dots + H_K [\vec{I}_{iK}] [\vec{I}_{mK}]^T \pmod{N} \end{aligned}$$

where

$g_{tim}$  (= vector  $g_{it}$  [vector  $I_{m1}$ ]) is a component corresponding to vector  $I_{m1}$  for entity  $m$ , selected from own vector  $g_{it}$  for 1st block of information specifying entity  $i$  ( $t = 0, 1, 2, \dots, T$ );

$x_{lim} = \text{vector } s_{il} [\text{vector } I_{ml}]$ ;

$x_{jim}$  (= vector  $s_{ij}$  [vector  $I_{mj}$ ]) is a component corresponding to vector  $I_{mj}$  for entity  $m$ , selected from own vector  $s_{ij}$  for  $j$ 'th block of information specifying entity  $i$  ( $j = 2, 3, \dots, K$ );

$K_{im}$  is a common key generated by one entity  $i$  for another entity  $m$ ; and

$y_{im}$  is sum of  $(K-1)$  components  $x_{jim}$  ( $j = 2, 3, \dots, K$ ), that is,  $y_{im} = x_{2im} + x_{3im} + \dots + x_{Kim}$ .